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THE GRAVITATIONAL FIELD OF AN ELLIPSOID

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In theory, parameters of a satellite orbit are computed from Kepler's laws. In practice, we also must deal with forces, which influence the satellite orbit and change it. We must take into account such factors as non-homogeneity of the Earth's masses, which results in a non-homogeneous gravitational field, electromagnetic field, gravitational perturbing forces of the Sun and the Moon, atmospheric and solar pressure.

Especially, it is important to estimate and make arrangements necessary to decrease influence of perturbing forces for navigational satellites. For example, the orbital motion of GLONASS satellites have no resonance with rotation of the Earth. This factor allows to diminish the influence of the non-centrality of the Earth's gravitational field on the satellite orbit. But we need to account for the influence of the gravitational field when correcting the satellite position in its orbital plane.

To estimate a precise value of influence of gravitational field, it must be recognized that the Earth has shape of an ellipsoid. This is an actual topic in geodetic research nowadays. Famous and successful research in this sphere is GRACE (Gravity Recovery and Climate Experiment) mission. Results of the investigations by this mission were to equip satellites of the next GOCE mission by space gradiometer and ultra-precise ion motor for orbit corrections, so that the height of flight could be kept within a range of ±5 meters. It is feasible to choose and keep the orbit at selected high order resonances and, by this way, to reach a maximum quality of products derived from gradiometer measurements.

The first step of my research is to describe the normal gravity potential U for an ellipsoid:

$$U(u,\beta) = \frac{kM}{E}\arctan\left(\frac{E}{u}\right) + \frac{\omega^2}{2}a^2\,\frac{q}{q_0}\left(sin^2\beta - \frac{1}{3}\right) + \frac{\omega^2}{2}(u^2+E^2)cos^2\beta, \tag{1}$$

where k is the universal gravitational constant, M is the mass of the Earth, $E = \sqrt{a^2 - b^2}$, a is the semi-major axis and b is the semi-minor axis, ω is angular velocity of the Earth;

$$q = \frac{1}{2}\left(\left(1 + 3\left(\frac{u}{E}\right)^2\right)\arctan\left(\frac{E}{u}\right) - 3\frac{u}{E}\right); \tag{2}$$

$$q_0 = \frac{1}{2}\left(\left(1 + 3\left(\frac{b}{E}\right)^2\right)\arctan\left(\frac{E}{b}\right) - 3\frac{b}{E}\right). \tag{3}$$

The next step is to compute the actual potential W including terms $n = 20$, $m = 20$, and then compute the difference $W - U$ for certain values. Necessary parameters are taken from the WGS 84 document. From the computational results we can distinguish a difference between gravitational field of sphere and ellipsoid, and conclude that taking into account shape of gravitational field is important to make precise estimation for satellite orbit correction.