RESEARCH OF EQUILIBRIUM STATE OF SPACE TETHER SYSTEM CONSIDERING ATMOSPHERIC DRAG

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The equilibrium equations of tether and subsatellite will be given with the consideration of tether mass, tether deformation and atmospheric drag. Space tether system equilibrium state will be obtained through solving the equilibrium equations of tether and subsatellite. In the part of simulation and analysis, two cases will be simulated. One case is that tether length is fixed and subsatellite mass is variable and the other is subsatellite mass is fixed and tether length is variable. The results of equilibrium state, tether tension and tether angle of deviation will be analysed and contrasted. Simulation results provide a reference for choosing the parameters of tether system in future space applications.

Key words atmospheric drag；space tether system；equilibrium state

1. Introduction

Because of the increase of human's activities range in atmosphere and the increasingly urgent understanding of atmospheric environment, meanwhile in order to adapt to modern atmospheric science research and satisfy the needs of national defense, aerospace, communication , etc, the study of atmospheric sounding technology has always been a research hotspot for national researchers. Due to the existence of atmospheric drag for atmosphere of 100 to 200 kilometers high, it is hard for traditional satellite to keep running in this height range. So it cannot obtain data effectively. However, atmosphere of this height range can be detected effectively by using space tether system equilibrium state. Owing to different payload masses and the height of payload, gravity gradient force and atmospheric drag effects on payload are different. So tether critical lengths of space tether system with various probes are also different. In order to make sure that space tether system with atmospheric probe can work regular at equilibrium state and avoid tether fracture, it is necessary to analyze and calculate the equilibrium of space tether system. Berryman J etc. [1] discuss the possibility of space tether system running on LEO with more than one payloads equilibrium state existence. Natarajan[2] studies on two-body space tether system stabilization on equilibrium state and analyzes three equilibrium states by using linear time-dependent dynamics model. Pettazzi, L. etc.[3] put forward a distributed navigation technology based on equilibrium configuration, which can make several satellites arrive at designated spots in space. Bensong Yu[4] mentions equilibrium state modeling ignoring various external perturbation factors. This paper analyses space tether system equilibrium state of different influence factors. Atmospheric drag and tether flexibility are taking into consideration in dynamics modeling.

2. Space Tether System Equilibrium State Analysis

Assumptions as follows are made during dynamics modeling.

(1) spacecraft runs in circular orbit and its mass is much bigger than satellite and tether under spacecraft.

(2) tether flexibility is considered.

(3) tether cross section is rounded.

(4) spacecraft motion is limited in orbital plane.

(5) payload shape is sphere. Payload radius can be ignored compared to tether length.

2.1 Tether Equilibrium State Analysis

As tether flexibility is considered, continuous medium model is adopted. Equation(1) can be obtained by force analysis on tether microelement.

$$\rho(S)\Delta S\frac{\partial^2 \boldsymbol{r}}{\partial t^2}=\boldsymbol{T}(S+\Delta S,t)-\boldsymbol{T}(S,t)+\boldsymbol{q}\Delta S \quad (1)$$

where $\boldsymbol{T}(S+\Delta S,t)-\boldsymbol{T}(S,t)$ is tether force increment, S is tether curve coordinate, $\boldsymbol{r}$ is tether microelement radius vector, t is time, $\boldsymbol{q}$ is distributed load.

Equation(1) divides ΔS and make $\Delta S\to 0$. Tether motion equation can be obtained.

$$\rho(S)\frac{\partial^2 \boldsymbol{r}}{\partial t^2}=\frac{\partial \boldsymbol{T}}{\partial S}+\boldsymbol{q} \quad (2)$$

Considering flexible tether cannot bear lateral load, tether force direction is along tangential.

$$\boldsymbol{T}=T\boldsymbol{\tau} \quad \boldsymbol{\tau}=\frac{1}{\gamma}\frac{\partial \boldsymbol{r}}{\partial S} \quad (3)$$

where $\boldsymbol{\tau}$ is unit tangent vector and $\gamma=\left|\frac{\partial \boldsymbol{r}}{\partial S}\right|$.

Tether equilibrium state condition can be obtained from tether motion equation.

$$\frac{d\boldsymbol{T}}{dS}+\boldsymbol{q}=0 \quad (4)$$

This equation is applied to both unstretchable tether and stretchable tether. In order to make reaerch results universal, stretchable tether is studied in this paper.

There are two kinds of distributed load $\boldsymbol{q}$ [5]: mass force such as gravity and surface force such as aerodynamic force.

Gravity is described as:

$$\boldsymbol{q}_{\mathrm{g}}=\frac{\mu}{\gamma(T)}g \quad (5)$$

where g is acceleration of gravity. μ is tether linear density. $\gamma(T)$ is tether radius rate of change.

Inertia force is described as:

$$\boldsymbol{q}_{\Phi}=-\frac{\mu}{\gamma(T)}\boldsymbol{a}_{\mathrm{e}} \quad (6)$$

where $\boldsymbol{a}_{\mathrm{e}}=\Omega^2 r$ is acceleration vector. Ω is space tether system orbit angular velocity. $r=\sqrt{x^2+y^2}$ is distance between tether microelement and the earth's core.

Aerodynamic force is described as:

$$\boldsymbol{q}_{\mathrm{R}}=-\frac{1}{2}C\rho_{\mathrm{a}}d_{\mathrm{T}}V\boldsymbol{V}\left|\sin\alpha\right| \quad (7)$$

where $\boldsymbol{V}$ is velocity tether microelement relative to atmosphere, C is aerodynamic drag coefficient, ρ_{a} is atmospheric density. $\alpha=\varphi-\theta$ is tether microelement angle of attack, d_{T} is tether diameter.

Research on space tether system equilibrium state is usually based on orbit motion coordinate system[5]. Interaction of tether microelement with distributed load is as Fig.1.

where τ is tether microelement tangential direction, n is tether microelement normal direction, q is distributed load interaction with tether microelement. O is the earth's core. $S1$ is spacecraft.

$$\frac{\mathrm{d}x}{\mathrm{d}S} = -\sin\theta \tag{8}$$

$$\frac{\mathrm{d}y}{\mathrm{d}S} = \cos\theta \tag{9}$$

where x, y are tether microelement coordinates.

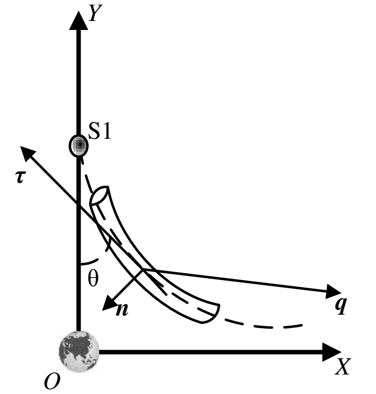

Fig. 1 Interaction of tether microelement with distributed load

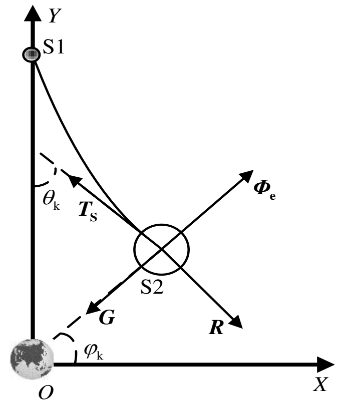

Fig. 2 Forces operating on the payload

Gravity, inertia force and aerodynamic force acting on tether microelement are projected on microelement tangent direction and normal direction. Equation(10) can be obtained combing Equation(4).

$$\frac{\mathrm{d}T}{\mathrm{d}S} = \frac{\mu}{\gamma(T)}(g - \Omega^2 r)\sin(\varphi - \theta) - q_{\mathrm{R}_\tau} \tag{10}$$

$$T\frac{\mathrm{d}\theta}{\mathrm{d}S} = \frac{\mu}{\gamma(T)}(\Omega^2 r - g)\cos(\varphi - \theta) - q_{\mathrm{R}_\mathrm{n}} \tag{11}$$

where $q_{\mathrm{R}_\tau} = \frac{1}{2}Cd_\mathrm{T}\rho_\mathrm{a}V^2 |\sin\alpha|\cos\alpha$, $q_{\mathrm{R}_\mathrm{n}} = -\frac{1}{2}Cd_\mathrm{T}\rho_\mathrm{a}V^2\sin^2\alpha$, $\sin\varphi = x/r$, $\cos\varphi = -y/r$.

Equation(8),(9),(10) and (11) are tether microelement equilibrium state equations.

2.2 Payload Equilibrium State Analysis

In order to obtain initial value condition to solve tether equilibrium state equations, payload needs to be force analyzed and its equilibrium state equations need to be built. Payload force situation is as Fig.2, where $S1$ is spacecraft. $S2$ is payload. O is the earth's core. $\boldsymbol{T}_\mathrm{S}$ is tether force. $\boldsymbol{G}$ is payload gravity. $\boldsymbol{R}$ is payload aerodynamic force. $\boldsymbol{\Phi}_\mathrm{e}$ is payload centrifugal force.

Payload is treated as mass point. Payload equilibrium state condition is as follows:

$$\boldsymbol{T}_\mathrm{S} + \boldsymbol{G} + \boldsymbol{R} + \boldsymbol{\Phi}_\mathrm{e} = 0 \tag{12}$$

Payload equilibrium state equations are as follows:

$$-(G - \Phi_\mathrm{e})\sin\varphi_\mathrm{k} - R\cos\varphi_\mathrm{k} + T_\mathrm{S}\cos\theta_\mathrm{k} = 0 \tag{13}$$

$$(G-\Phi_e)\cos\varphi_k - R\sin\varphi_k + T_S\sin\theta_k = 0 \qquad (14)$$

where $G=\dfrac{mK}{r_k^2}$, $\Phi_e = m\Omega^2 r_k$, $R = C_S\dfrac{\rho_a(r_k)V^2}{2}S_m$. r_k is payload center of mass radius vector. m is payload mass. K is earth gravitational constant. Ω is spacecraft orbit angular velocity. C_S is payload front drag coefficient. S_m is payload cross sectional area. V is payload linear velocity relative to atmosphere. Equations(13),(14) are payload equilibrium state equations.

3. Equilibrium State Calculation and Simulation

Simulation parameters are as follows: $K=3.986\times10^5\text{km}^3/\text{s}^2$, the earth mean radius: $R_3 = 6371.02\text{km}$, spacecraft orbital altitude: $H_0 = 300\text{km}$, tether linear density: $\mu = 0.1667\text{kg/km}$, tether radius: $r_0 = 3\times10^{-4}\text{m}$ tether Young modulus: $E = 2.5\times10^{10}\text{N/m}^2$, tether drag coefficient: $C = 2.2$, payload front drag coefficient: $C_S = 2$.

3.1 Equilibrium State Calculation and Analysis when Tether Length is fixed

Tether length is selected as $S = 150\text{km}$.

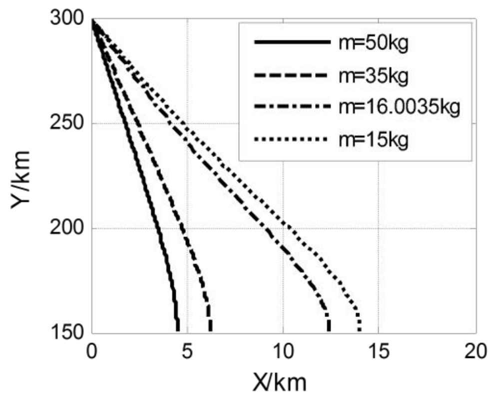

Fig. 3 Equilibrium position when tether length $S = 150\text{km}$

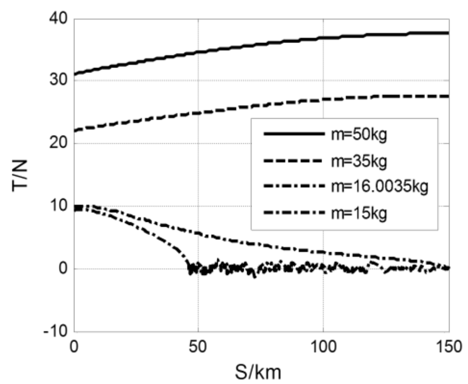

Fig. 4 Relationship between tether tension and natural coordinates when tether length $S = 150\text{km}$

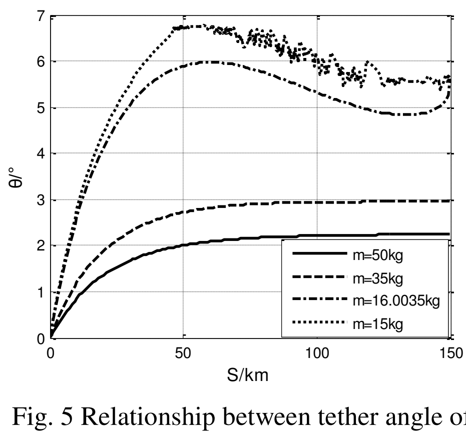

Fig. 5 Relationship between tether angle of deviation and natural coordinates when tether length $S = 150\text{km}$

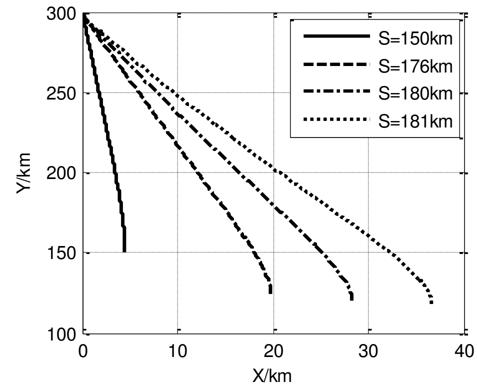

Fig. 6 Equilibrium position when subsatellite mass $m = 50\text{kg}$

It can be seen from the simulation result Fig.3 that the horizontal distance between subsatellite and spacecraft increases along with subsatellite mass decreases when the system reaches equilibrium position. From Fig.4 it can be seen that tether tension of the junction point of subsatellite gradually decreased along with subsatellite mass decreases. Tether tension variation trend which is from the junction point of subsatellite to the junction point of spacecraft changes from increasing to decreasing. When subsatellite mass $m = 16.0035\text{kg}$, tether tension of the junction point of spacecraft reaches zero. When subsatellite mass $m < 16.0035\text{kg}$, tether tension appears negative value. It means tether is loosed. Fig. 5 shows variation trend of tether angle of deviation. It can be seen that when subsatellite mass gets smaller, it is more influenced by atmospheric drag and may cause tether loosed.

3.2 Equilibrium State Calculation and Analysis when Subsatellite Mass is fixed

Subsatellite mass is selected as $m = 50\text{kg}$

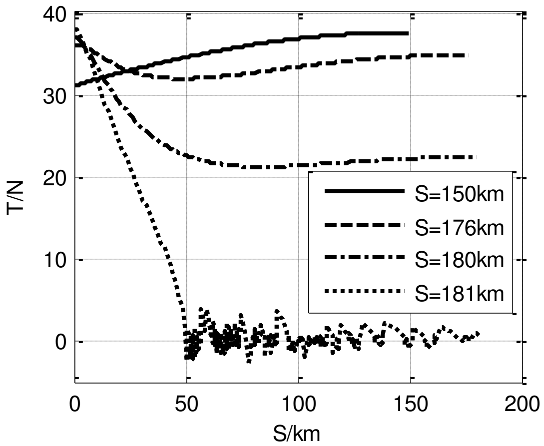

Fig. 7 Relationship between tether tension and natural coordinates when tether length $m = 50\text{kg}$

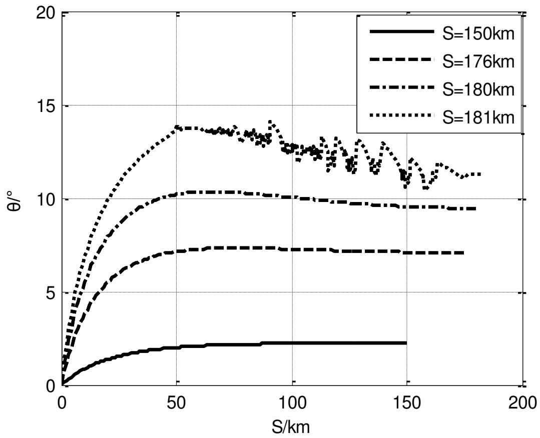

Fig. 8 Relationship between tether angle of deviation and natural coordinates when tether length $m = 50\text{kg}$

It can be seen from Fig.6 that distance between subsatellite and spacecraft increases along with tether length increases when the system reaches equilibrium position. From Fig.7 it can be seen that tether tension of the junction point of subsatellite gradually increased along with tether length increases. Tether tension variation trend which is from the junction point of subsatellite to the junction point of spacecraft changes from increasing to decreasing. When tether length $S = 181\text{km}$, tether tension appears negative value. It means tether is loosed. Fig. 8 shows variation trend of tether angle of deviation. It can be seen that when tether length gets longer, it is more influenced by atmospheric drag and may cause tether loosed.

4. Conclusion

Tether and payload equilibrium state equations are built in this paper by analyzing space tether system equilibrium state influenced by atmospheric drag in this paper. Space tether system equilibrium state is obtained by calculating Tether and payload equilibrium state equations. It is obtained by simulation that when tether length is fixed, tether will get loosed if subsatellite mass larger than its critical value. When subsatellite mass is fixed, tether will get loosed if tether length longer than its critical length.

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